https://dx.doi.org/10.22161/ijaers.5.1.12 ISSN: 2349-6495(P) | 2456-1908(O)

The Creation of a Vortex in Sea water through the MHD

Osvaldo Missiato[1], Celso Luis Levada[2], Alexandre Luis Magalhães Levada[3]

[1]Universities Integrated Einstein of Limeira- Brazil
[2]Hermínio Ometto Foundation- Uniararas- Brazil
[3]Federal University of São Carlos- Brazil

***Abstract*— *The Magneto hydrodynamics equations, MHD, are discussed for weakly conductive fluids of electricity, and then an analytical solution is presented. An MHD vortex is generated using seawater in a cylindrical vessel, where external electromagnetic fields are conveniently applied. Some physical quantities of interest are discussed and measured by well-known techniques.*

Keywords— MHD, Magneto hydrodynamics equations, cylindrical vessel, Vortex.

I. INTRODUCTION

Weak electrically conductive fluids, when subjected to electromagnetic forces, may respond somewhat surprisingly depending on how the force in the fluid volume applies.

Even though the fluid is weakly conductive, the density of electromagnetic force, $\vec{f} = \vec{j} \times \vec{B}$; being $\vec{j}$ the current density and B the externally applied magnetic field may be sufficient to induce large movements. For this, it is necessary that the magnetic field be of relatively great intensity, because the currents that are induced in these, in general, are very small since the electrical conductivity is relatively small, as indicated by BRAGINSKII(1).

The rotationality of the magnetic force, that is, $\nabla \times \vec{f} \neq \vec{0}$ is critical to generate fluid movement and thus create MHD vortex.

In the experiment, the fluid is considered to be at rest at the instant the electromagnetic force (Lorentz force) is applied. The establishment of the vortex in the fluid occurs after a few seconds of force application and is visualized quite clearly. The vorticity, given by the rotational velocity vector, is a physical quantity of great interest and is associated with the concept of angular velocity. A rotational speed meter, called a measurement rotor, was introduced into a vessel containing the liquid in rotational motion. By means of the stroboscopic technique, it was possible to measure the angular velocity. Values of the order of 1000 RPM are common for magnetic fields of the order of 0.3 Tesla and applied voltage of the order of 400 V/m.

It is possible to emphasize the importance of the Hartmann layer in the mechanism of generation of the rotational movement of the fluid; the electric current tends to circulate in the mentioned layer giving the maximum force applied, since in this case the field is axial and forms an angle of 90º with the current intensity.

THE IMPORTANCE OF VORTICITY

Consider an incompressible conducting fluid subjected to the influence of external electromagnetic fields, initially at rest. Consider, also, the second law of Newton per unit volume, expressed by:

$$\rho \vec{a} = -\nabla p + \vec{f} \qquad (1)$$

where ρ is the specific mass, $\vec{a}$ is the acceleration vector, p is the pressure and $\vec{f}$ is the force field. Applying the rotational operator on equation (1), the pressure p and the force $\vec{f}$ are mathematically eliminated if it is irrotational or conservative. If the force $\vec{f}$ is rotational (viscous or Lorentz), i.e. if it has the tendency to induce a rotation about the fluid element or to modify its rotational state, then a path for solving the fluid motion may be found in function of the vorticity $\vec{\omega}$. It should be noted that the vorticity $\vec{\omega}$ is a measure of the average rotation at each point of the fluid. It can also be shown that the movement of a fluid of uniform mass, occupying a closed vessel, does not occur without the presence of vorticity (MOFFAT(2)).

II. FAILED DRIVER FLUIDS : BASIC EQUATIONS

According to LEWELLEN(3), the equations that describe the behavior of an incompressible conductive fluid, subject to the influence of external electromagnetic fields in the low frequency regime, where the displacement current is neglected, are given by:

$$\nabla \cdot \vec{V} = 0 \qquad (2)$$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \vec{g} + \vec{j} \times \vec{B} + \eta \nabla^2 \vec{V} \qquad (3)$$

https://dx.doi.org/10.22161/ijaers.5.1.12 ISSN: 2349-6495(P) | 2456-1908(O)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (4)$$

$$\vec{j} = \frac{1}{\mu} \nabla \times \vec{B} \quad (5)$$

$$\vec{j} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right) \quad (6)$$

where $\vec{V}$ is the velocity vector, p is the pressure, ϱ is the specific mass, $\vec{J}$ is the current density, $\overrightarrow{B_o}$ is the magnetic field, $\vec{E}$ is the electric field, σ is the electrical conductivity, η is the viscosity and μ is the magnetic permeability.

Consider, however, that

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla \quad (7)$$

Let $R_m = \mu\sigma UL$ be the Magnetic Reynolds number, which is defined by the ratio between convection and diffusion of the magnetic field by the fluid. On the other hand, using equations (4), (5) and (6), it can be shown that

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{V} \times \vec{B}\right) + \eta \nabla^2 \vec{B}, \eta = \frac{1}{\mu\sigma}$$

And so it comes that

$$Rm = \frac{\left|\nabla \times \left(\vec{V} \times \vec{B}\right)\right|}{\left|\eta \nabla^2 \vec{B}\right|} = \mu\sigma UL$$

In this development, U is a characteristic velocity and L is a characteristic length. Considering that $\mu = 4\pi 10^{-7} SI$ and σ ≈4S/m, we find that in this work we have $R_m << 1$, since U≈ 1m/s and L≈ 1m. On the other hand, by theory, we know that although the fluid is initially at rest, the force $\vec{f} = \vec{j} \times \vec{B}$ imposes a movement on it. Then, due to this movement, an induced electromotive force of the order of VB is generated in the fluid, as well as a current of magnitude equal to σVB. The magnetic field B_1, produced by this current, is approximately equal to the product of the magnetic Reynold number, Rm, by the externally applied magnetic field, B. That is, $B_1 \approx R_m.B$. In contrast, considering that $R_m << 1$, we can say that B_1 is negligible. Analogous reasoning can be established for the induced electric field, whose value E1 is given by the product of Rm by the module of the velocity vector V, by the module of the external magnetic field B, that is, $E_1 \approx R_m VB$. Again, because $R_m <<1$, this induced electric field is also discarded. Thus, the resulting magnetic field is the applied external magnetic field itself. Since, by hypothesis, B is assumed to be uniform, Landau shows that ∇XB = 0 and that ∇.B = 0.

It is noted, then, that the Law of Ampere does not apply, so that the electric currents are determined by the Law of Ohm. Thus, the Lorentz force is given by

$$\vec{f} = \vec{j} \times \vec{B} = \left[\sigma\left(\vec{E} + \vec{V} \times \vec{B}\right) \times \vec{B}\right] \quad (8)$$

III. SIMPLIFIED SOLUTION

A very simple solution of the problem is one in which, in a cylindrical coordinate system (r, Θ, z), only the azimuth velocity V_Θ is considered, making $V_r = V_z = 0$.

The justification for such a hypothesis is based on the application of a strong external magnetic field on the fluid. Thus, everything happens as if the fluid were suspended in the applied field.

It is a completely different situation from a sink vortex, for example, where radial and axial velocities are considerable.

Assuming the flow as stationary and with axial symmetry,

$$\left(\frac{\partial}{\partial t} = 0\right) \quad e \quad \left(\frac{\partial}{\partial \theta} = 0\right)$$

Besides that, it is considered that $E_z = E_\Theta = 0$; $B_z = B_o \approx$ constant.

Thus, the system of equations proposed in the previous section becomes:

$$j_r \cong \sigma Er \quad ; \quad j_\theta = 0 \quad (9)$$

$$-\frac{V_\theta^2}{r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \quad (10)$$

$$\rho g + \frac{\partial p}{\partial z} = 0 \quad (11)$$

$$\frac{\eta}{\rho}\left(\frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta^2}{r}\right) - \frac{1}{\rho} Bo j_r = 0 \quad (12)$$

The conservation of the electric charge or continuity of the electric current ensures that

$$\frac{1}{r}\frac{\partial}{\partial r}(r j_r) = 0 \quad (13)$$

With this we have that rJr is constant.

Taking into account that rJr is constant, in equation (12) and considering that

$$J = \frac{xjr}{\sigma Bo V_{\theta_o}}$$

e

$$x = \frac{r}{r_o}, V = \frac{V_\theta}{V_{\theta_o}}, V^* = \frac{E}{Bo V_{\theta_o}}, H = Bor_o\left(\frac{\sigma}{\eta}\right)^{1/2}$$

We arrive at the dimensionless form of equation (12), given by:

$$x^2 \frac{\partial^2 V}{\partial x^2} + x \frac{\partial V}{\partial x} - V = xH^2 J \tag{13}$$

The resolution of this linear and second-order differential equation is

$$V(x) = C_1 x + \frac{C_2}{x} + \frac{H^2 J}{2} x \ln x \tag{14}$$

The last term of equation (14) represents the electromagnetic driving force. The integration constants C1 and C2 are determined by the boundary condition given by:

1) $V(x=1) = 0$
2) $V(x=x_0) = V_0 \approx 0$, pois $x = \frac{r}{r_0} << 1$

Substituting the boundary conditions 1) and 2) in equation (13), we find

$$C_1 = \frac{H^2 j}{2} x_o^2 \ln x_o \tag{15}$$

$$C_2 = -\frac{H^2 j}{2} x_o^2 \ln x_o \tag{16}$$

It can be seen that the current density is related to the potential difference applied through the quantities explained by Eq. (9). In addition, taking into account the definition of V*, and making the necessary transformations[4], we have

$$V^* = \frac{1}{r_o B o V_{\theta_o}} \int_{x_o}^{1} E_r d_r = J \ln x_o \tag{17}$$

On the other hand, considering that $\vec{\omega} = \nabla \times \vec{V}$, taking the relations of Eq. (10), expressions for pressure and vorticity are easily found.

IV. EXPERIMENTAL PART

In the specialized literature, there are few experimental works on the generation, measurement and analysis of vortexes by magneto hydrodynamics. Even so, most of them, like SOMMERIA[5], treat with relatively high electrical conductivity fluids, such as mercury, sodium and various liquid metal alloys, where the effects of induction play a relevant role and many approaches made in this paper do not apply. Then, with the purpose of adding something more relevant to the mentioned literature, we will present next an experiment that deals with the generation of a forced vortex, using as a working fluid the sea water and electromagnetic fields chosen in a convenient way.

DESCRIPTION OF THE EXPERIMENT

In a cylindrical vessel, shown in fig. 1, two electrodes are fitted, one of which, of gold, is installed in the center of the vessel, is hemispherical 4 mm in diameter. The other is of stainless steel in ring format of variable diameter and concentric to the first. That is, sea water is added to the height of approximately 10cm. The cylinder with water rests on the basis of a solenoid of 50cm in diameter so as to benefit to the maximum from the intensity of the applied magnetic field which on average is of the order of 0.35 Tesla and is located very close to the center of the system. The solenoid is powered by a source, generating a current of the order of 90 A. Another electrical device provides an adjustable voltage that is applied between the two gold electrodes and stainless steel to establish a potential difference and hence an electric current in the liquid. The applied electric field ranges from zero to approximately 1000V / m.

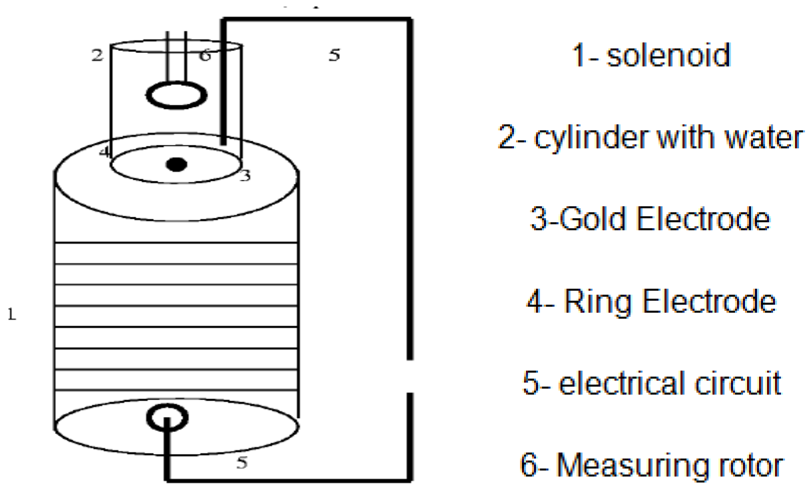


Fig.1: Experimental apparatus

https://dx.doi.org/10.22161/ijaers.5.1.12 ISSN: 2349-6495(P) | 2456-1908(O)

SOME FINDINGS

When only the electric field is acting, there is no movement of the liquid, however, at the instant it is applied, also, the magnetic field, starts to act the magnetic force. In this case, such force acts in the tangential direction and accelerates the fluid. In contrast, the Hartmann layer acts in order to conduct the electric current more easily in the radial direction, contributing to make the tangential force more intense. As can be deduced from the texts of SHERCLIFF and HUNT(6), under the conditions mentioned in the previous paragraph, an increase in the electric current density occurs. An experimental verification of the current circulation by the Hartmann layer can be inferred by moving the ring-shaped electrode along the axis of symmetry. It is noted(7), therefore, that the vortex continues to be formed and loses little intensity even when the ring-shaped electrode is displaced vertically.

V. RESULTS

Experiment-1

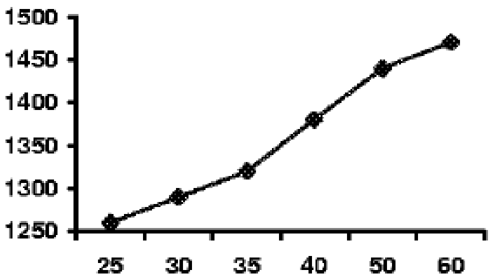
In this case, for a pair of gold and brass electrodes, the following values were used: Fixed magnetic field, equal to B = 0.39T Rotor - R = 2.9 cm; Radius = 5.25 cm; Fluid column in cylinder - 6cm of water column; Rotor apparatus placed 1.5 cm from the surface; Room temperature.

In this way, the data of table 1

ΔU (v)	Ω (c/s)	Ω (R.P.M.)
25	21	1260
30	21,5	1290
35	22	1320
40	23	1380
50	24	1440
60	24,5	1470

Tabela 1 – Valores de ΔU, Ω(Hz),Ω(RPM).

The values of ΔU in volts and Ω, in RPM, allowed the sketch of graph 1.



Graph 1 of ΔU (Volt) and Ω (RPM).

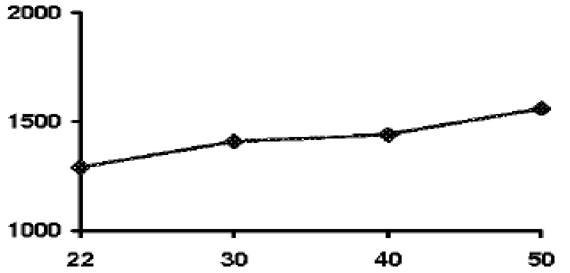
Experiment 2:

Magnetic field B = 0,39Tlas; water level 6cm; Radius = 5,25cm, with the measuring device 1.5cm from the surface; Temperature = 70C. In view of this, the data in table 2

ΔU (v)	Ω (c/s)	Ω (R.P.M.)
22	21,5	1290
30	23,5	1410
40	24,0	1440
50	24,0	1560

Table 2 - Values of ΔU (V), Ω (Hz), Ω (RPM).

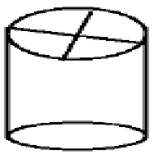
The values of ΔU in volts and Ω, in RPM, allowed the sketch of graph 2.



Graph 2 of ΔU (Volt) and Ω (RPM).

Experiment 3:

Measurement of vortex rotation using cork stopper; Magnetic field B = 0.36 Tesla; Gold and brass electrode; Container with 5cm of water column; Ring electrode placed on the bottom of the vessel; Radius = 5.25 cm



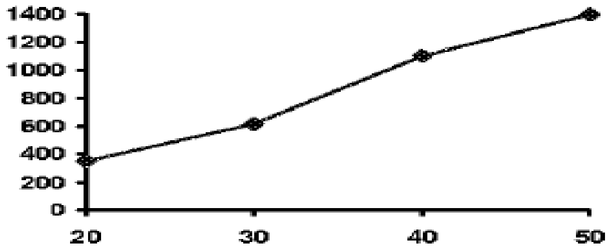
cork stopper

Therefore, the data presented in table 3

ΔU (v)	Ω (R.P.M.)
20	350
30	615
40	1100
50	1400

Table 3 - Values of ΔU (V) and Ω (RPM)

The values of ΔU in volts and Ω, in RPM, allowed the sketch of graph 3.



Graph 3 of ΔU (Volt) and Ω (RPM).

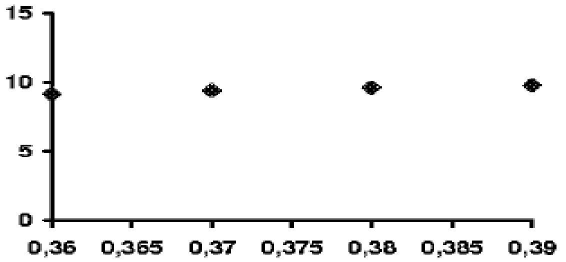
Experiment 4:

Keeping the voltage constant and varying the magnetic field. By doing this, we obtained the data from table 4

TENSÃO		VALORES MÉDIOS (CORRENTE)	CAMPO B
$\Delta U = 30V$	$R_1 = 9{,}14\Omega$	$I = 3{,}28$ A	$B_1 = 0{,}376$ T
$\Delta U = 30V$	$R_2 = 9{,}43\Omega$	$I = 3{,}18$ A	$B_2 = 0{,}365$ T
$\Delta U = 30V$	$R_3 = 9{,}64$	$I = 3{,}11$ A	$B_3 = 0{,}375$ T
$\Delta U = 30V$	$R_1 = 9{,}80\Omega$	$I = 3{,}06$ A	$B_4 = 0{,}39$ T

Table 4 - Values of voltage, resistance, current and magnetic field

From table 4 was constructed the graph n.4 referring to the values of the electric resistance as a function of the magnetic field.



Graph 4 - Electrical resistance as a function of the magnetic field.

VI. CONCLUSIONS

This work shows that seawater behaves in a somewhat surprising way as MHD working fluid. It can be concluded that the increase of the magnetic field intensity and the reduction of the electric current are of extreme importance for the increase of the speed of rotation and also of the efficiency of the experiment, since, the reduction of the electrical current causes the decrease of the Joule effect. The obtaining of new data, point by point, with more refined techniques, will be necessary to be able to compare the results with the proposed theoretical model and to change it in a compatible way, if it is the case. It was also verified an increase of the electric resistance as a function of the increase of the applied magnetic field. Theoretically, as SOMMERIA[5] indicates, the formula for resistance is:

$$R = \frac{Bo}{2\pi\sqrt{\sigma\eta}} \ln\left(\frac{Ro}{a}\right)$$

Since the constant a represents the radius of the core of the vortex and R0 corresponds to the radius of the cylindrical vessel.

Finally, some comments can be made between the theoretical model and the experimental part.

1. In the flow core, at first approximation, the viscosity can be neglected. Thus, by the Work - Energy theorem, we have that the velocity at a point of the fluid is given by:

$$V = \left(\frac{4\pi\sigma|\Delta\Phi|B_o}{\rho}\right)^{1/2}$$

Considering that:

$$\sigma = 4{,}3\,\frac{S}{m}$$

$|\Delta\emptyset| = 160V$ a 1,2 cm from the center of the container

$B_0 = 0{,}3$ Teslas $\qquad \rho = 1030\,\frac{kg}{m^3}$

The measured value of velocity V = 1.5 m / s, which can be compared with the theoretical value given by V = ωr, is obtained.

Taking the mean value for the angular velocity ω ≈ 1300RPM and, where r = 1.2 cm, we reached the value V = 1.6 m / s, being very close to the value measured experimentally.

At the core of the flow, the electric current tends to flow through the Hartmann layer, so we have j≈0, so it turns out that

$$\nabla\Phi' \approx \vec{V}\times\vec{B} \quad \text{e} \quad V \approx \frac{1}{B}\frac{\Delta\Phi}{\Delta r}$$

Substituting the velocity value V = 1.6 m / s and using a mean magnetic field value equal to 0.3 Teslas

$$\frac{\Delta\emptyset}{\Delta r} \approx 0{,}5\,\frac{V}{m}$$

That is within the expected value.

REFERENCES

[1] BRAGINSKII, S.I. Magneto hydrodynamics of Weakly Conducting Liquids, Soviet Physics, JETP, v.37; n.5, 1960

[2] MOFFAT, H.K. Some problems in the Magneto hydrodynamics of Liquids Metal, ZAMM, v.58; 1978

[3] LEWELLEN, W.S.; Magneto hydrodinamically Driven Vortices, Proc. Heat Transf. Fluid Mech. Inst. Stanford University Press, v.1, 1960

[4] LANDAU,L. &LIEFCHITZ, E.; Electro Dynamics of Continuous Media, v.VIII, Ed.Mir, 1969.

[5] SOMMERIA,J.; Electrically Driven Vortices in a Strong Magnetic Field, J.Fluid Mech.v.189, 1988

https://dx.doi.org/10.22161/ijaers.5.1.12 *ISSN: 2349-6495(P) | 2456-1908(O)*

[6] HUNT, J.C. & SHERCLIFF, J.A., 1971, Magneto hydrodynamics at High Hartmann Number, Ann. Rev. Fluid Mech., vol. 3, pp. 37-62.
[7] SHERCLIFF, J. A., 1970, Fluid Mechanics due to a electric current source, J. Fluid Mech., vol. 40, part 2, pp. 241-250.